Théorie des goupilles de raquette

Théorie avancée des goupilles de raquette

Cas d'une absence de glissement entre spiral et goupille

Caractéristiques du spiral avec une spire externe semi-circulaire

Référence :E:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Dimensions $ép = 0.03\,mm$ $ha = 0.15\,mm$ $S = 4.5 \times 10^{-3}\,mm^2$

$d2_{sp} = 4.52\,mm$ $d1_{sp} = 1.1\,mm$ $p_{sp} = 0.135\,mm$ $n_{sp} = 12.667$

$L_{sp} = 11.182\,cm$ $\psi_0 := 2\cdot\pi\cdot n_{sp}$ $\psi_0 = 4.56 \times 10^3\,deg$ $E = 2.093 \times 10^{11}\,m^{-2}N$

Position du piton $r_P := 0.5\cdot d_{piton}$ $\alpha_P := 0$ $x_P := r_P\cdot cos(\alpha_P)$ $y_P := r_P\cdot sin(\alpha_P)$

$x_P = 2.55\,mm$ $y_P = 0\,mm$ $z_P := x_P + i\cdot y_P$

Position du point d'attache à la virole $r_V := 0.5\cdot d1_{sp}$ $\alpha_V(\theta) := \psi_0 + \theta$ $x_V(\theta) := r_V\cdot cos(\alpha_V(\theta))$ $y_V(\theta) := r_V\cdot sin(\alpha_V(\theta))$

Position du point de raccordement sur le spiral $\alpha_A := 180\cdot deg$ $r_A := 0.5\cdot d2_{sp}$ $z_A := r_A\cdot e^{i\cdot\alpha_A}$

Spire externe formée d'un demi-cercle $R_0 := r_P$ $x_{0t}(\alpha_t) := R_0\cdot cos(\alpha_t)$ $y_{0t}(\alpha_t) := R_0\cdot sin(\alpha_t)$ $z_{0t}(\alpha_t) := R_0\cdot e^{i\cdot\alpha_t}$

$s_t(\alpha_t) := R_0\cdot\alpha_t$ $l_t := s_t(\alpha_A)$ $l_t = 8.011\,mm$

Forme initiale du spiral

$a := \dfrac{p_{sp}}{2\cdot\pi}$ $r_s(\alpha) := r_A - a\cdot(\alpha - \alpha_A)$ $x_{0s}(\alpha) := r_s(\alpha)\cdot cos(\alpha)$ $y_{0s}(\alpha) := r_s(\alpha)\cdot sin(\alpha)$

$s_s(\alpha) := \dfrac{1}{2\cdot a}\cdot\left(r_A{}^2 - r_s(\alpha)^2\right)$ $s_s(\alpha) := r_A\cdot(\alpha - \alpha_A) - \dfrac{a}{2}\cdot(\alpha - \alpha_A)^2$ $L_t := s_s(\psi_0 + \alpha_A) + l_t$

$L_t = 11.983\,cm$

Position angulaire des goupilles par rapport au piton:

$\varepsilon := 0.06$ $s_g := \varepsilon\cdot L_t$ $s_g = 7.19\,mm$ $\alpha_g := \dfrac{s_g}{R_0}$ $\alpha_g = 161.548\,deg$

Amplitude stationnaire du balancier $\theta_0 = 270\,deg$

Position radiale de la goupille pour une élongation de contact donnée

Première approximation de la déformée de la spire externe entre piton et goupille $\theta_1 := 20\cdot deg$

$x_{0g}(\alpha_g) := R_0\cdot cos(\alpha_g)$ $y_{0g}(\alpha_g) := R_0\cdot sin(\alpha_g)$

$\xi_{0g}(\alpha_g) := \dfrac{R_0}{\alpha_g}\cdot sin(\alpha_g)$ $\eta_{0g}(\alpha_g) := \dfrac{R_0}{\alpha_g}\cdot(1 - cos(\alpha_g))$

$$\beta := arctan\left[\frac{-(x_{0g}(\alpha_g) - \xi_{0g}(\alpha_g))}{y_{0g}(\alpha_g) - \eta_{0g}(\alpha_g)}\right] + \pi$$

Jeu entre spiral et goupille au repos $j := \dfrac{\varepsilon\cdot(y_{0g}(\alpha_g) - \eta_{0g}(\alpha_g))}{cos(\beta)}\cdot\theta_1$ $j = 0.06\,mm$

Calcul de la matrice $\boldsymbol{D_g}$

Calcul à partir de la première approximation de la déformée

$$z_{1t}(\theta, \alpha_t) := z_P + \frac{L_t \cdot R_0}{L_t + \theta \cdot R_0} \cdot \left(exp\left(i \cdot \alpha_t \cdot \frac{L_t + \theta \cdot R_0}{L_t} \right) - 1 \right)$$

$x_1(\theta, \alpha) := Re(z_{1t}(\theta, \alpha))$ $\quad$ $y_1(\theta, \alpha) := Im(z_{1t}(\theta, \alpha))$ $\quad$ $EI := E \cdot I_s \cdot N^{-1} \cdot m^{-2}$ $\quad$ $EI = 7.063 \times 10^{-8}$

$$\xi_{1g}(\alpha_g) := \frac{1}{\alpha_g} \cdot \int_0^{\alpha_g} x_1(\theta_0, \alpha)\, d\alpha \qquad \eta_{1g}(\alpha_g) := \frac{1}{\alpha_g} \cdot \int_0^{\alpha_g} y_1(\theta_0, \alpha)\, d\alpha$$

$$p2_{1g}(\alpha_g) := \frac{1}{\alpha_g} \cdot \int_0^{\alpha_g} x_1(\theta_0, \alpha)^2\, d\alpha \qquad q2_{1g}(\alpha_g) := \frac{1}{\alpha_g} \cdot \int_0^{\alpha_g} y_1(\theta_0, \alpha)^2\, d\alpha$$

$$k_{1g}(\alpha_g) := \frac{1}{\alpha_g} \cdot \int_0^{\alpha_g} x_1(\theta_0, \alpha) \cdot y_1(\theta_0, \alpha)\, d\alpha$$

$d1_{11}(\alpha_g) := q2_{1g}(\alpha_g) \cdot m^{-2}$ $\quad$ $d1_{22}(\alpha_g) := p2_{1g}(\alpha_g) \cdot m^{-2}$ $\quad$ $d1_{33}(\alpha_g) := 1$ $\quad$ $\mathrm{R}_0 := R_0 \cdot m^{-1}$

$d1_{12}(\alpha_g) := -k_{1g}(\alpha_g) \cdot m^{-2}$ $\quad$ $d1_{13}(\alpha_g) := \eta_{1g}(\alpha_g) \cdot m^{-1}$ $\quad$ $d1_{23}(\alpha_g) := -\xi_{1g}(\alpha_g) \cdot m^{-1}$

$$\boldsymbol{D_{1g}}(\alpha_g) := \frac{\mathrm{R}_0 \cdot \alpha_g}{EI} \cdot \begin{pmatrix} d1_{11}(\alpha_g) & d1_{12}(\alpha_g) & d1_{13}(\alpha_g) \\ d1_{12}(\alpha_g) & d1_{22}(\alpha_g) & d1_{23}(\alpha_g) \\ d1_{13}(\alpha_g) & d1_{23}(\alpha_g) & 1 \end{pmatrix} \qquad \boldsymbol{D_{1g}}(\alpha_g) = \begin{pmatrix} 0.277 & -0.035 & 152.033 \\ -0.035 & 0.277 & -26.646 \\ 152.033 & -26.646 & 1.018 \times 10^5 \end{pmatrix}$$

Approximation à partir de la forme naturelle du spiral

$$p2_{0g}(\alpha_g) := \frac{R_0^2}{2 \cdot \alpha_g} \cdot \left(\alpha_g + \frac{sin(2 \cdot \alpha_g)}{2} \right) \qquad q2_{0g}(\alpha_g) := \frac{R_0^2}{2 \cdot \alpha_g} \cdot \left(\alpha_g - \frac{sin(2 \cdot \alpha_g)}{2} \right) \qquad k_{0g}(\alpha_g) := \frac{R_0^2}{2 \cdot \alpha_g} \cdot sin(\alpha_g)^2$$

$d_{11}(\alpha_g) := q2_{0g}(\alpha_g) \cdot m^{-2}$ $\quad$ $d_{22}(\alpha_g) := p2_{0g}(\alpha_g) \cdot m^{-2}$ $\quad$ $d_{33}(\alpha_g) := 1$

$d_{12}(\alpha_g) := -k_{0g}(\alpha_g) \cdot m^{-2}$ $\quad$ $d_{13}(\alpha_g) := \eta_{0g}(\alpha_g) \cdot m^{-1}$ $\quad$ $d_{23}(\alpha_g) := -\xi_{0g}(\alpha_g) \cdot m^{-1}$

$$\boldsymbol{D_g}(\alpha_g) := \frac{\mathrm{R}_0 \cdot \alpha_g}{EI} \cdot \begin{pmatrix} d_{11}(\alpha_g) & d_{12}(\alpha_g) & d_{13}(\alpha_g) \\ d_{12}(\alpha_g) & d_{22}(\alpha_g) & d_{23}(\alpha_g) \\ d_{13}(\alpha_g) & d_{23}(\alpha_g) & 1 \end{pmatrix} \qquad \boldsymbol{D_g}(\alpha_g) = \begin{pmatrix} 0.366 & -0.012 & 179.392 \\ -0.012 & 0.296 & -29.138 \\ 179.392 & -29.138 & 1.018 \times 10^5 \end{pmatrix}$$

Calcul des formes quadratiques

$Delta(\alpha_g) := sin(\alpha_g)$ $\quad$ $\gamma(\alpha_g) := cos(\alpha_g)$ $\quad$ $x_{0g}(\alpha_g) := R_0 \cdot cos(\alpha_g)$ $\quad$ $y_{0g}(\alpha_g) := R_0 \cdot sin(\alpha_g)$

$$H(\alpha_g) := \frac{R_0}{\alpha_g} \cdot (1 - cos(\alpha_g)) \qquad J(\alpha_g) := \frac{R_0}{\alpha_g} \cdot (\alpha_g - sin(\alpha_g))$$

$$\boldsymbol{V_1}(\alpha_g) := \begin{pmatrix} \gamma(\alpha_g) & Delta(\alpha_g) & 0 \end{pmatrix}^T \qquad \boldsymbol{V_2}(\alpha_g) := \begin{pmatrix} Delta(\alpha_g) & -\gamma(\alpha_g) & -\mathrm{R}_0 \end{pmatrix}^T$$

$$W1_{g1}(\alpha_g) := \frac{1\cdot s^2\cdot kg^{-1}}{2}\cdot \boldsymbol{V_1}(\alpha_g)^T\cdot \boldsymbol{D_{1g}}(\alpha_g)\cdot \boldsymbol{V_1}(\alpha_g) \qquad W1_{g1}(\alpha_g) = 0.149\ kg^{-1}\cdot s^2$$

$$W_{g1}(\alpha_g) := \frac{R_0{}^3}{4\cdot E\cdot I_s}\cdot(\alpha_g - sin(\alpha_g)\cdot cos(\alpha_g)) \qquad W_{g1}(\alpha_g) = 0.183\ kg^{-1}\cdot s^2$$

$$W1_{g2}(\alpha_g) := \frac{1\cdot s^2\cdot kg^{-1}}{2}\cdot \boldsymbol{V_2}(\alpha_g)^T\cdot \boldsymbol{D_{1g}}(\alpha_g)\cdot \boldsymbol{V_2}(\alpha_g) \qquad W1_{g2}(\alpha_g) = 0.4\ kg^{-1}\cdot s^2$$

$$W_{g2}(\alpha_g) := \frac{R_0{}^3}{4\cdot E\cdot I_s}\cdot\left[3\cdot\alpha_g + sin(\alpha_g)\cdot(cos(\alpha_g) - 4)\right] \qquad W_{g2}(\alpha_g) = 0.405\ kg^{-1}\cdot s^2$$

$$W1_{g3}(\alpha_g) := \frac{1\cdot s^2\cdot kg^{-1}}{2}\cdot \boldsymbol{V_1}(\alpha_g)^T\cdot \boldsymbol{D_{1g}}(\alpha_g)\cdot \boldsymbol{V_2}(\alpha_g) \qquad W1_{g3}(\alpha_g) = 0.209\ kg^{-1}\cdot s^2$$

$$W_{g3}(\alpha_g) := \frac{R_0{}^3}{4\cdot E\cdot I_s}\cdot(1 - cos(\alpha_g))^2 \qquad W_{g3}(\alpha_g) = 0.223\ kg^{-1}\cdot s^2$$

$$V_g(\alpha_g) := \begin{pmatrix} R_0\cdot sin(\alpha_g) & -R_0\cdot cos(\alpha_g) & 1\end{pmatrix}^T$$

$$W1_{c2}(\alpha_g) := \frac{1\cdot kg\cdot m^2\cdot s^{-2}}{2}\cdot\left(\left(V_g(\alpha_g)\right)\right)^T\cdot \boldsymbol{D_{1g}}(\alpha_g)^{-1}\cdot V_g(\alpha_g) \qquad W1_{c2}(\alpha_g) = 2.358\times 10^{-5}\ m^2\cdot kg\cdot s^{-2}$$

$$\Delta_g(\alpha_g) := \frac{R_0{}^4}{4\cdot\alpha_g{}^3}\cdot(\alpha_g - sin(\alpha_g))\cdot\left[\alpha_g\cdot(\alpha_g + sin(\alpha_g)) - 4\cdot(1 - cos(\alpha_g))\right] \qquad \Delta_g(\alpha_g) = 1.237\,mm^4$$

$$W_{c2}(\alpha_g) := \frac{E\cdot I_s\cdot R_0{}^3}{8\cdot\Delta_g(\alpha_g)\cdot\alpha_g{}^3}\cdot\left[3\cdot\alpha_g{}^2 - 2\cdot\alpha_g\cdot sin(\alpha_g)\cdot(2 + cos(\alpha_g)) - (1 - cos(\alpha_g))\cdot(1 - 7\cdot cos(\alpha_g))\right]$$

$$W_{c2}(\alpha_g) = 3.741\times 10^{-5}\ m^2\cdot kg\cdot s^{-2}$$

Coefficient de frottement limite

$$F1(\alpha_g) := \frac{H(\alpha_g)\cdot W1_{g3}(\alpha_g) - J(\alpha_g)\cdot W1_{g1}(\alpha_g)}{H(\alpha_g)\cdot W1_{g2}(\alpha_g) - J(\alpha_g)\cdot W1_{g3}(\alpha_g)} \qquad F1(\alpha_g) = 0.131$$

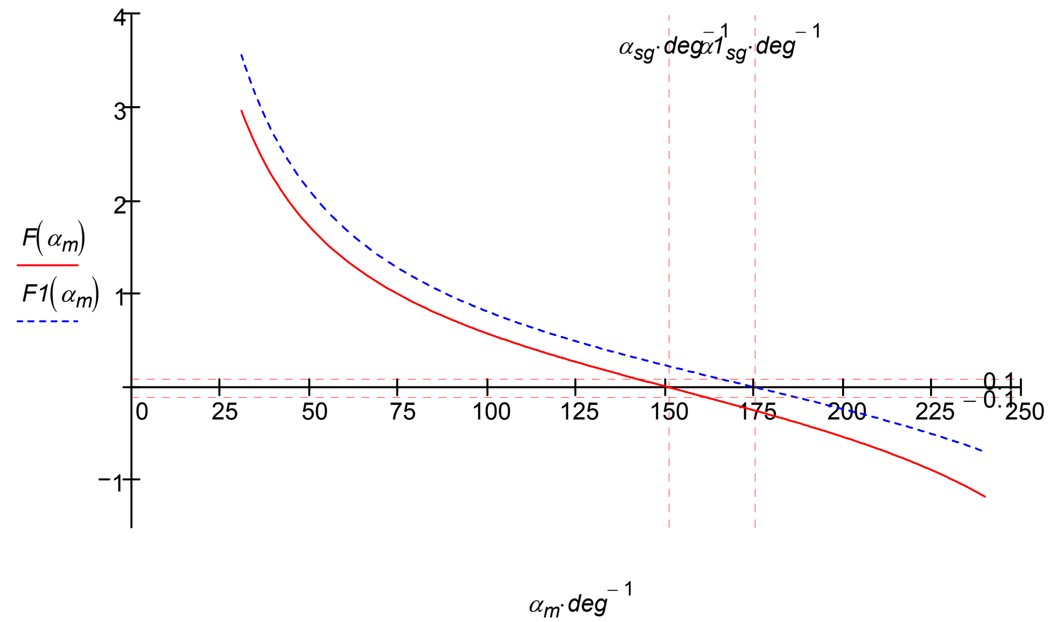
$$F(\alpha_g) := \frac{H(\alpha_g)\cdot W_{g3}(\alpha_g) - J(\alpha_g)\cdot W_{g1}(\alpha_g)}{H(\alpha_g)\cdot W_{g2}(\alpha_g) - J(\alpha_g)\cdot W_{g3}(\alpha_g)} \qquad F(\alpha_g) = -0.104$$

$$F(\alpha_g) := \frac{-\alpha_g{}^2 + \alpha_g\cdot sin(\alpha_g)\cdot(1 + cos(\alpha_g)) + (1 - cos(\alpha_g))\cdot(1 - 3\cdot cos(\alpha_g))}{(1 - cos(\alpha_g))\cdot\left[(2 + cos(\alpha_g))\cdot\alpha_g - 3\cdot sin(\alpha_g)\right]} \qquad F(\alpha_g) = -0.104$$

$\alpha_{sg} := 151\cdot deg \qquad \alpha_{sg} := racine(F(\alpha_{sg}), \alpha_{sg}) \qquad \alpha_{sg} = 151.277\,deg$

$\alpha 1_{sg} := 175\cdot deg \qquad \alpha 1_{sg} := racine(F1(\alpha 1_{sg}), \alpha 1_{sg}) \qquad \alpha 1_{sg} = 175.718\,deg$

$\alpha_m := 31\cdot deg, 32\cdot deg\,..\,240\cdot deg$



Réaction normale à la goupille

Coefficient de frottement

$$\mu_g := 0.1$$

$$F1_{gn}(\theta, \alpha_g) := \frac{\varepsilon \cdot H(\alpha_g) \cdot (\theta - \theta_1)}{2 \cdot (W1_{g1}(\alpha_g) - \mu_g \cdot W1_{g3}(\alpha_g))} \qquad F1_{gn}(\theta_0, \alpha_g) = 1.8 \times 10^{-3}\, N$$

$$F_{gn}(\theta, \alpha_g) := \varepsilon \cdot (\theta - \theta_1) \cdot \frac{2 \cdot E \cdot I_s}{R_0^{\,2}} \cdot \frac{1 - \cos(\alpha_g)}{\alpha_g \cdot \left[\alpha_g - \sin(\alpha_g) \cdot \cos(\alpha_g) - \mu_g \cdot (1 - \cos(\alpha_g))^2\right]}$$

$$F_{gn}(\theta_0, \alpha_g) = 1.434 \times 10^{-3}\, N$$

Accroissement de l'angle polaire de la tangente au spiral au point de contact

$$N1(\alpha_g) := \frac{E \cdot I_s}{2 \cdot R_0 \cdot \alpha_g \cdot W1_{c2}(\alpha_g)} \qquad N1(\alpha_g) = 0.208$$

$$N(\alpha_g) := \frac{E \cdot I_s}{2 \cdot R_0 \cdot \alpha_g \cdot W_{c2}(\alpha_g)} \qquad N(\alpha_g) = 0.131 \qquad \varphi_{sg}(\theta, \alpha_g) := \varepsilon \cdot (\theta - \theta_1) \cdot (N(\alpha_g) - 1)$$

$$\Delta\varphi_g := \varepsilon \cdot (\theta_0 - \theta_1) \cdot N(\alpha_g) \qquad \Delta\varphi_g = 1.969\, deg \qquad \Delta\varphi_g := \varepsilon \cdot (\theta_0 - \theta_1) \cdot N1(\alpha_g) \qquad \Delta\varphi_g = 3.125\, deg$$

Sans goupille

$$\Delta\varphi := \theta_0 \cdot \frac{R_0 \cdot \alpha_g}{L_t} \qquad \Delta\varphi = 16.2\, deg$$

Perturbation de marche dans le cas de deux goupilles

$$\phi_1(\theta_1, \theta_0) := \arcsin\left(\frac{\theta_1}{\theta_0}\right) \cdot (-\theta_0 \le \theta_1 \le \theta_0) + \frac{\pi}{2} \cdot (\theta_1 > \theta_0) - \frac{\pi}{2} \cdot (\theta_1 < -\theta_0)$$

$$\phi_2(\theta_2, \theta_0) := -\arcsin\left(\frac{\theta_2}{\theta_0}\right) \cdot (-\theta_0 \le \theta_2 \le \theta_0) + \frac{\pi}{2} \cdot (\theta_2 < -\theta_0) - \frac{\pi}{2} \cdot (\theta_2 > \theta_0)$$

$$\delta_1(\theta_1, \theta_0) := \frac{-\varepsilon}{4 \cdot \pi} \cdot (\pi - 2 \cdot \phi_1(\theta_1, \theta_0) - \sin(2 \cdot \phi_1(\theta_1, \theta_0)))$$

$$\delta_2(\theta_2, \theta_0) := \frac{-\varepsilon}{4 \cdot \pi} \cdot (\pi - 2 \cdot \phi_2(\theta_2, \theta_0) - \sin(2 \cdot \phi_2(\theta_2, \theta_0)))$$

Spiral à égales distances des goupilles en position d'équilibre: $\theta_2 := -\theta_1$

Théorie élémentaire

$\delta_1(\theta_1, \theta_0) = -0.014$ $\delta_2(\theta_2, \theta_0) = -0.014$ $\phi_1(\theta_1, \theta_0) = 0.074$ $\phi_2(\theta_2, \theta_0) = 0.074$

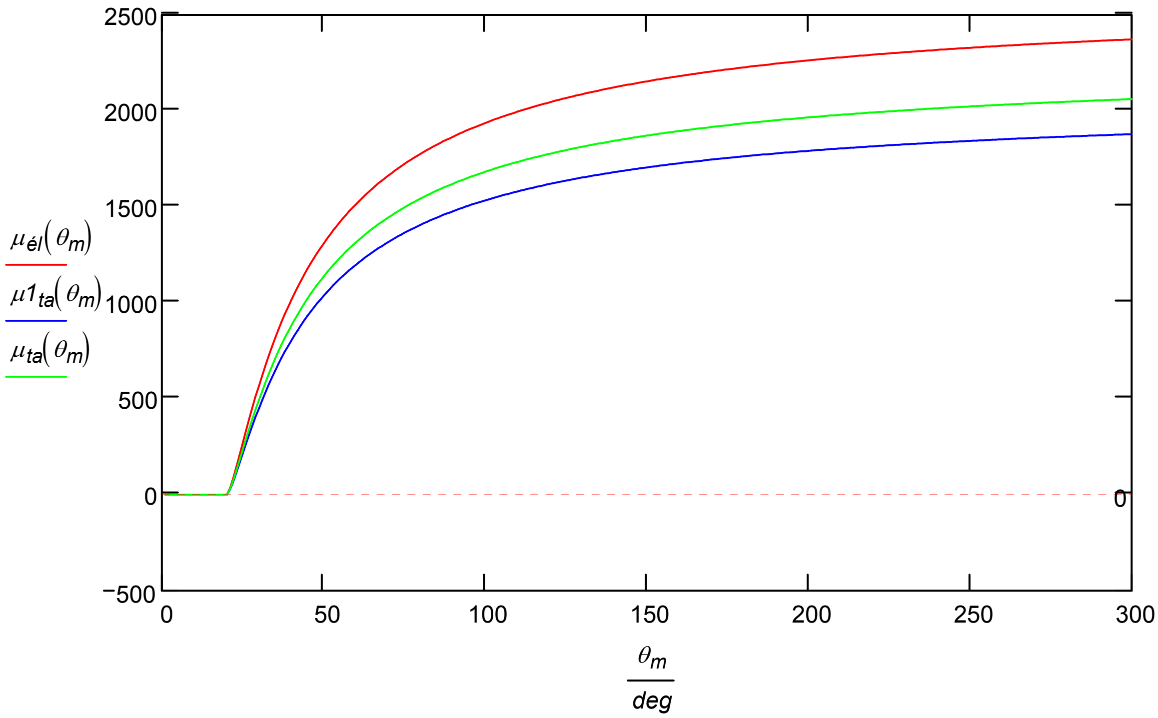
$\mu_1(\theta_0, \theta_1) := -86400 \cdot (\delta_1(\theta_1, \theta_0) + \delta_2(\theta_2, \theta_0))$ $\mu_1(\theta_0, \theta_1) = 2.348 \times 10^3$ $\mu_{él}(\theta_0) := \mu_1(\theta_0, \theta_1)$

Théorie avancée

$$\mu 1_{ta}(\theta_0) := \mu_1(\theta_0, \theta_1) \cdot (1 - N1(\alpha_g))$$

$$\mu_{ta}(\theta_0) := \mu_1(\theta_0, \theta_1) \cdot (1 - N(\alpha_g))$$

$$\theta_m := 1 \cdot deg, 2 \cdot deg .. 300 \cdot deg$$



Réglage par la raquette

$$l(\alpha_g) := R_0 \cdot \alpha_g$$

$$\delta_1(\theta_1, \theta_0, \alpha_g) := \frac{-1}{4 \cdot \pi} \cdot \frac{l(\alpha_g)}{L_t - l(\alpha_g)} \cdot (1 - N(\alpha_g)) \cdot (\pi - 2 \cdot \phi_1(\theta_1, \theta_0) - \sin(2 \cdot \phi_1(\theta_1, \theta_0)))$$

$$\mu_{ta}(\theta_1, \theta_0, \alpha_g) := -86400 \cdot (2 \cdot \delta_1(\theta_1, \theta_0, \alpha_g))$$

$$\alpha_m := 60 \cdot deg, 70 \cdot deg .. 180 \cdot deg$$